DII-003-016403 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

May / June – 2015

Mathematics : Course No. 4003 : (Number Theory - 2)

(New Course)

Faculty Code : 003
Subject Code : 016403

Time : 150 Minutes] [Total Marks : **70**

Instructions: *(1) There are five questions.*
(2) All questions are compulsory.
(3)Each question carries 14 marks.

Q.1 Fill in the blanks: (Each question carries two marks)

(i) If a/b and c/d are consequetive Farrey fractions in the n^{th} row then _______is the unique rational between a/b and c/d with the smallest denominater.

(ii) If r is a rational multiple of π then the sum of all rational values of $\cos r\pi$=_________

(iii) If m and n are positive integers and θ is a rational solution of x^n=m then θmust be____________

(iv) If x_0 is a positive integer such that 1+2+3+…+x_0=y^2 then(2x_0+1,y) is a solution of Pell's equation______________.

(v) If the continued fraction expansionof θ is periodic then θ= __________.

(vi) If θ >1 and $\theta + \theta^{-1} < \sqrt{5}$ then θ <________ and θ^{-1}>_____________

(vii) If θ is an irrational number and a/b is a rational number with b>0,(a,b)=1 and $|\theta - a/b| < \frac{1}{2b^2}$ then a/b=____________

Q.2 Attempt any two.

(a) State and Prove Hurwitz's Theorem using Farrey Fraction Method. 7

(b) If θ is an irrational number and $\frac{h_n}{k_n}$ denotes the nth convergent corresponding to the continued fraction expansion of θ then prove that $|\theta k_{n-1} - h_{n-1}| > \frac{1}{k_{n+1}} > |\theta k_n - h_n|$,$\forall$ n $\geq$1. 7

(c) If $a_0,a_1,a_2,\ldots,a_n$are positive integers and $x > 1$then prove that$<a_0,a_1,a_2,\ldots a_{n-1},x> = \frac{xh_{n-1}+h_{n-2}}{xk_{n-1}+k_{n-2}}$. Deduce that $< a_0,a_1,\ldots a_n > = \frac{h_n}{k_n}$ for all n$\geq$ 1 7

Q.3 All are compulsory:

(a) Suppose $a_0, a_1, a_2, \ldots, a_n$are positive integers and $\frac{h_n}{k_n}$ are the nth convergents corresponding to this intergers then prove that for all $n \geq 1$, $h_n k_{n-2} - h_{n-2} k_n = (-1)^n$. 5

(b) Prove that every irrational number θ can be uniquely expressed as an infinite simple continued fraction. 6

(c) Suppose $\theta$is an irrational number, $\frac{a}{b}$ is a rational number with b>0 and$\frac{h_n}{k_n}$are the nth convergent for θ.If $|\theta b - a| < |\theta k_n - h_n|$ implies $b \geq k_{n+1}$then prove that $\left|\theta - \frac{a}{b}\right| < \left|\theta - \frac{h_n}{k_n}\right|$ implies $b \geq k_n$. 3

OR

Q.3 All are compulsory:

(a) Prove that if θ is a rational multiple of π and cos θ is rational then $|cos\theta|$ cannot be different from 0 ,$\frac{1}{2}$ and 1. 7

(b) Suppose (x_1,y_1) is the smallest positive solution of $x^2-dy^2=1$(d is a positive integers with is not a perfect square) .Prove that all the positive solutions of the above equaition are given by $x_n + \sqrt{d}\, y_n = (x_1 + \sqrt{d}\, y_1)^n$ (n = 1,2,3,..). 7

Q.4 Attempt any two of the following:

(a) Prove that the Diophantine equation $15x^2-7y^2 = 9$ has no solution in integers. 7

(b) Suppose (x,y,z) is a primitive Pythagorean triplet. Prove that there are positive integers r and s such that r>s , (r,s) = 1 and $x= r^2- s^2$, y= 2rs and $z = r^2 + s^2$. 7

(c) Prove that for every integer $n \geq 1$ there is a polynomial $f_n(x)$ with integer coefficients and leading coefficient 1such that $f_n(2cos\theta) = 2cosn\theta$ for all θ. 7

Q.5 Do as directed: (Each question carries two marks)

(i) Find the simple continued fraction expansion of $\sqrt{2}$ -1.

(ii) Find the simple continued fraction expansion of $\frac{1741}{31}$.

(iii) Find the irrational number θ whose continued fraction expansion is $< 4,4,8,4,8,4,8,\ldots>$.

(iv) Find the primitive pythagorean tripletes for which z < 60.

(v) Find first three positive solutions of $x^2 - 19y^2 = 1$.

(vi) Find the simple continued fraction expansion of $\frac{1}{\sqrt{2}}$.

(vii) Find first three positive solutions of $x^2 - 2y^2 = -1$.